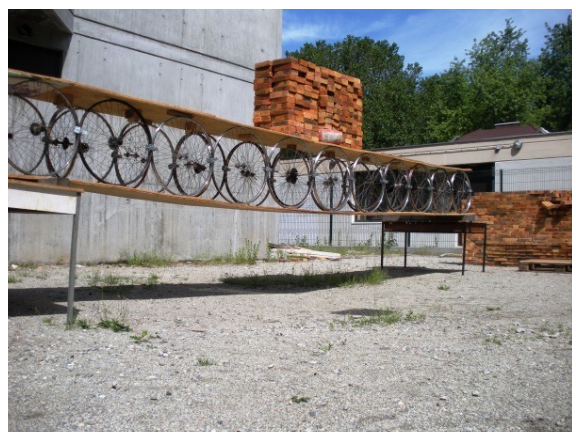
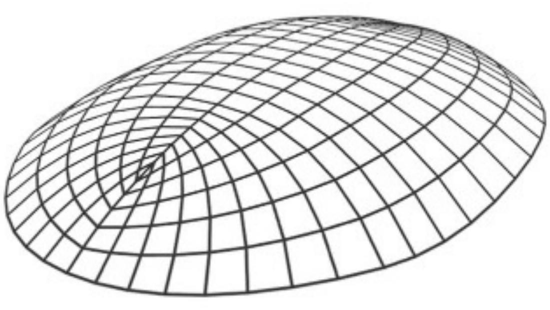
# Make complex structures affordable

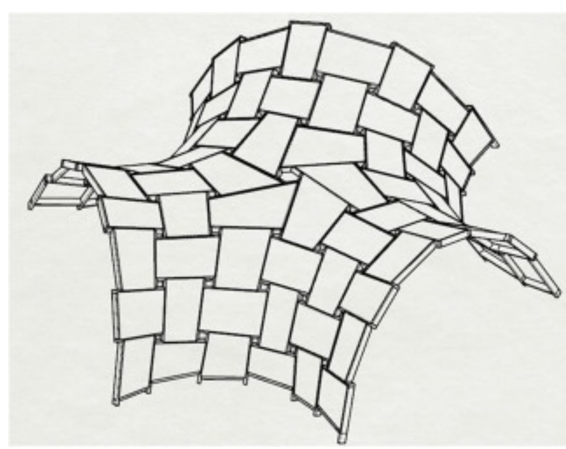
Olivier Baverel
Ecole des Ponts ParisTech / Ecole nationale superieure d'architecture de Grenoble

## Abstract

The presentation focuses on research proposals that make complex structures more affordable constructions. By reinforcing the links between mechanics of materials, structural engineering, applied mathematics and life cycle analysis, new paths may be explored and building innovations proposed. First a focus is made on the life cycle analysis showing that CO2 is not the only parameter to tackle environmental problems. Secondly proposals based on a rigorous mathematical management of shapes and geometry to rationalize complex situations in a fully integrative way, including cladding and connections will be explained. These knowledges may help to find ways to reuse manufactured goods to limit the environmental impact of construction, many case studied will be shown. Finally, new digital and technological tools such as robot or 3D printing allow to revisit ancient techniques and to generate innovative solutions.







# Similarity-driven topology finding of surface patterns in architecture

Robin Oval
Delft University of Technology, The Netherlands

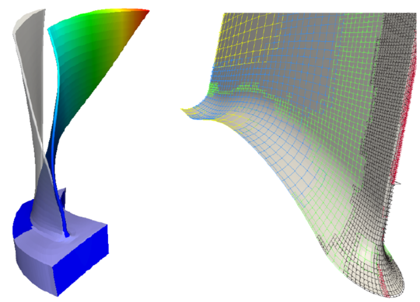
## Abstract

Structural design is a search for the best trade-off between multiple architecture, engineering, and construction objectives, not only mechanical efficiency or construction rationality. Producing hybrid designs from single-objective optimal designs to explore multi-objective trade-offs is common in the design of structural forms, constrained to a single parametric design space. However, producing topological hybrids offers a more complex challenge, as a combinatorial problem that is not encoded as a finite set of real numbers but as an unbonded series of grammar rules. This presentation will focus on a strategy for the generation of hybrid designs of quad-mesh pattern topologies for surface structures. Based on a quad-mesh grammar, an algebra is introduced to measure the distance between designs, find their similar features, and enumerate designs with different degrees of topological similarity. To achieve this, the operators of topological distance, intersection and union of quad meshes will be defined. Structural design applications will be shown to highlight the use of topologically hybrid designs as a surrogate for obtaining multi-objective trade-offs.

# Efficient Matrix Assembly and Adaptive Refinement in Isogeometric Analysis

Bert Jüttler,
Johannes Kepler University

**Abstract**



Isogeometric Analysis is a computational framework for numerical simulation, which was introduced by T.J.R. Hughes et al. in 2005 with the aim of bridging the gap between Design and Analysis, by adopting the prevailing mathematical technology of tensor product splines for discretizing of partial differential equations (PDEs). This presentation will address two of the many challenges that arise in this context. First, while the use of spline discretizations clearly offers advantages in terms of the number of degrees of freedom required compared to classical finite elements, these advantages are then compromised by the higher computational cost of matrix assembly in isogeometric analysis. We describe our methods for efficient matrix assembly, which make use of spline projection, pre-computed look-up tables and sum factorization to optimize the computational performance of the entire process. Second, since the rigid structure of tensor product splines is an obstacle to the use of adaptive refinement in isogeometric analysis, various generalizations of them have been proposed in the literature. These include T-splines (introduced by Sederberg et al. in 2003), hierarchical B-splines (invented by Forsey and Bartels in 1988) and the so-called "locally refined" splines (Dokken et al. 2013). In this presentation, we will analyze these approaches and compare them with the truncated variant of hierarchical B-splines, which reconciles the requirements of isogeometric analysis with those of geometric design.

## References

[1] C. Giannelli, B. Jüttler, S. K. Kleiss, A. Mantzaflaris, B. Simeon, J. Špeh, "THB-splines: An effective mathematical technology for adaptive refinement in geometric design and isogeometric analysis", Comput. Meth. Appl. Mech. Engrg., vol. 299, pp. 337–365, 2016.

[2] M. Pan, B. Jüttler, A. Giust, "Fast formation of isogeometric Galerkin matrices via integration by interpolation and look-up", Comput. Meth. Appl. Mech. Engrg. vol. 366 (2020), 113005.

# Discrete conformality and beyond.
# Where geometry meets computer graphics and mathematical physics

Alexander I. Bobenko
Institute of Mathematics, Technische Universität Berlin, Germany


## Abstract

Structure-preserving discretization in the field of geometry is the paradigm of discrete differential geometry. In some aspects, the discrete theory turns out to be even richer than its smooth counterpart. It focuses on developing constructive methods. The well-established theory of discrete conformal maps and circle patterns is related to discrete integrable models of mathematical physics and has found applications in geometry processing. We present their generalizations beyond the conformal limit: decorated discrete conformal maps [1, 2] and ring patterns [3, 4], which share the corresponding existence and uniqueness statements. The theory and construction methods are based on convex variational principles related to hyperbolic geometry. We define discrete constant mean curvature (cmc) surfaces (soap bubble surfaces) [5] in terms of sphere packings with orthogonally intersecting circles. These discrete cmc surfaces can be constructed from orthogonal ring patterns. The data used for the construction is purely combinatorial - the combinatorics of the curvature line pattern. Numerous virtual and printed models as well as animation movies will be demonstrated.

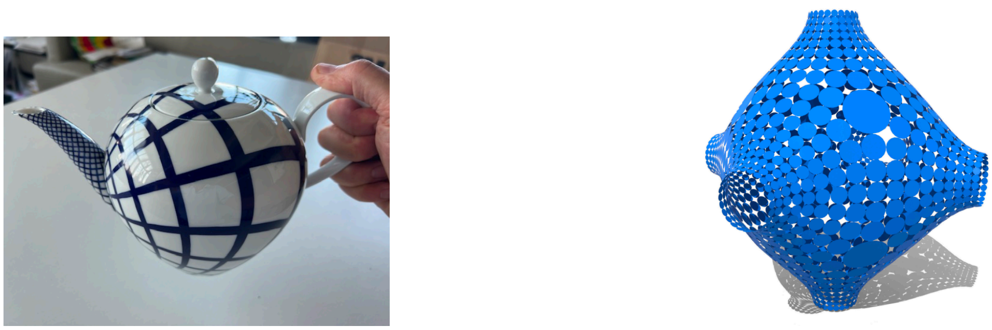



Figure 1: Left: Conformally parametrized tea pot costructed using discrete conformal mappings.
Right: A discrete cmc surface constructed using orthogonal ring patterns [5].

## References


[1] A.I. Bobenko, C.O. Lutz, Decorated discrete conformal maps and convex polyhedral cusps, Intern. Math. Research Notices 2024:12 (2024), 9505-0534, doi.org/10.1093/imrn/rnae016

[2] A.I. Bobenko, C. Lutz, Decorated discrete conformal equivalence in non-Euclidean geometries (2023) arXiv:2310.17529 [math.GT]

[3] A.I. Bobenko, T. Hoffmann, T. Rörig, Orthogonal ring patterns in the plane, Geometria Dedicata (2023), doi.org/10.1007/s10711-023-00859-y

[4] A.I. Bobenko, Spherical and hyperbolic orthogonal ring patterns: integrability and variational principles (2024) arXiv:2409.06573 [math.MG] [math.GT]

[5] A.I. Bobenko, T. Hoffmann, N. Smeenk, Constant mean curvature surfaces from ring patterns: Geometry from combinatorics (2024) arXiv:2410.08915 [math.DG]

# Discretization of quadrics and of elliptic coordinates

Yuri B. Suris
Institut für Mathematik, Technische Universität Berlin, Germany

**Abstract**

In this talk, I will review a recently found discretization of classical elliptic coordinate systems. These systems became prominent after Jacobi used them for integrating several famous problems of classical mechanics, including the two centers problem and the geodesics on an ellipsoid. A structure preserving discretization of these coordinate systems remained open for a long time and was finally tackled in Refs. [1, 2]. I will closely follow the history of this discovery. On the first step [1], a construction based on an integrable discretization of the Euler-Poisson-Darbox equation was used. The coordinate functions of the resulting discrete nets are given in terms of gamma functions. These nets enjoy separability property, their two-dimensional subnets being Koenigs nets with an additional novel discrete analog of the orthogonality property (thus, discrete isothermic, in a sense). On the second step [2], the novel orthogonality concept was put at the very basis of a more general construction. The latter is geometric, via polarity with respect to a sequence of classical confocal quadrics. The coordinate functions of discrete confocal quadrics were computed explicitly. This opens the possibility to close the cycle of historic development by applying discrete elliptic coordinate systems to discretize corresponding problems in classical mechanics in the structure preserving fashion.

## References

[1] A.I. Bobenko, W. Schief, Yu.B. Suris, J. Techter. On a discretization of confocal quadrics. I. An integrable systems approach. *J. Integrable Systems*, 2016, **1**, No. 1, xyw005, 34 pp.

[2] A.I. Bobenko, W. Schief, Yu.B. Suris, J. Techter. On a discretization of confocal quadrics. II. A geometric approach to general parametrizations. *Internat. Math. Research Notices*, 2020, **2020**, No. 24, 10180-10230.

# Surface Rationalization and Optimization in Structural Engineering Practice

Toby Mitchell
Thornton Tomasetti, Chicago, United States

### Abstract

In this talk, we examine the nuances and challenges of deploying the techniques of architectural geometry and discrete differential geometry in commercial structural engineering practice. By contrast to the academic context, structural design of surface structures in building practice involves multiple overlapping optimization objectives, many of which are not easily quantifiable, and many of which are not fully clear at the outset of the design process. Instead, the particular mathematical techniques appropriate for a given design problem must be uncovered by engineers working together with architects and other specialists in an iterative design process that integrates structural and construction performance goals with other technical objectives as well as subjective design intent. The author will examine approaches that have proven successful in his work on practical engineering projects, such as

1. The use of graphic statics in the design of the flat-paneled negatively-curved grid shell of the Hangzhou Greenland Center (which recently won the CTBUH's Best Tall Building award in the Asian region [1]).

2. The use of the Airy stress function in structural design of the flat-paneled quad-dominant grid shell of the Columbus, Ohio John Glenn airport renovation, and the utility of self-Airy shells in incorporating multiple panelization objectives while retaining structural performance [2].

3. The use of the static-kinematic duality in the design of rigidly-foldable structural origami such as SOM's MAK pavilion, and in the design of doubly-curved flat-paneled cable nets [3]

In addition to discussing the specific mathematical techniques used in these projects, the author will focus on the practical aspects of deploying architectural geometry and structural form-finding in design practice, such as the need to educate engineers on specialized techniques that are not typically part of their education, the need to solicit buy-in and facilitate authorship of architectural designers, the phasing of project development that necessitates the use of "lightweight" mathematical methods that do not rely on extensive information that will not be available in the early phases of design development, and the need to respond to input from contractors which often comes at the very end of a design process and may necessitate substantial design revisions.

## References

[1] "SOM Wins Seven Awards of Excellence from the Council on Tall Buildings and Urban Habitat", https://www.som.com/news/ctbuh-awards-24/.

[2] Cameron Millar, Toby Mitchell, Arek Mazurek, Ashpica Chhabra, Alessandro Beghini, Jeanne N. Clelland, Allan McRobie, and William F. Baker, On designing plane-faced funicular gridshells, Int. J. Space Structures, vol. 38, issue 1 (2022), https://doi.org/10.1177/09560599221126656.

[3] Toby Mitchell, Arek Mazurek, Christian Hartz, Masaaki Miki, and William F. Baker, Structural Applications of the Graphics Statics and Static-Kinematic Dualities: Rigid Origami, Self-Centering Cable Nets, and Linkage Meshes, Proceedings of IASS Annual Symposia, IASS 2018 Boston Symposium: Graphic statics, pp. 1-8(8)

# Variable Projection (VarPro) Method and Form-finding of Tension-compression Mixed Shells

Masaaki Miki
The University of Tokyo, Japan

## Abstract

This presentation reviews recent advances in the form-finding of tension-compression mixed shells. Although purely compressive stress states are traditionally considered ideal for shell structures, we propose that allowing a mix of tension and compression can expand the range of feasible shell geometries. The key challenge lies in the fact that the equilibrium problem becomes a hyperbolic boundary value problem, which is notoriously difficult to solve. We point out that the introduction of the Airy's stress function reveals that the equilibrium equation is a bilinear partial differential equation (PDE). We then indicate that this PDE can be solved using the Variable Projection (VarPro) method—developed specifically for bilinear problems. We also demonstrate that the alignment of stress and curvature directions is governed by a bilinear PDE, which can be solved concurrently with the equilibrium equation using the VarPro method.

# Local & Global Property Quantification With Persistent Homology

R. U. Gobithaasan
School of Mathematical Sciences, Universiti Sains Malaysia, Penang, Malaysia.

Kenjiro T. Miura
Graduate School of Science and Technology, Shizuoka University, Hamamatsu, Shizuoka, Japan.

### Abstract

Topological Data Analysis (TDA) is a powerful algebraic topology framework that aims to understand the shape and structure of complex datasets, particularly those with high dimensionality point cloud data $X \in R^d$ [1]. It has been successfully used for various types of Machine Learning tasks [2, 3]. Persistent Homology (PH), the main methodology in TDA, quantifies the shape and structure of complex datasets by representing the topological dan geometrical information of data in the form of Persistence Diagram denoted as $D_k(X)$. A $D_k(X)$ consisting of a set of 2-tuple $(b_i, d_i) \in R^2$, corresponds to a pairing between the births of $k^{th}$ homology class at $b_i$ and its death at $d_i$ along the filtration of $X$. $D_k(X)$ can be converted in the form of vector spaces that can be directly used as features for machine learning (ML) pipelines. It is known that topological features manifest as long-lived birth-death pairs in the $D_k(X)$, indicating their presence across multiple spatial scales. Recently it was found that $(b_i, d_i)$ close to diagonal encodes the geometrical feature of $X$ [4, 5]. The first part of this talks delves the law of composition [6] which makes an art beautiful and its relation to the types of shape analysis tools developed; hence leading to the development of Persistent Homology. We will then review the variety of framework for capturing geometrical and topological features across different spatial scales for understanding the underlying structure and relationships within the data. Overall, this talk provides insight into the implementation of PH framework not just as ML tasks, but also for the development of visually pleasing products.

## References

[1] G. Carlsson, "Topology and data", Bull. Amer. Math. Soc. **46**, 255–308, (2009).

[2] R. U. Gobithaasan, Zabidi Abu Hasan, et al., "Clustering Selected Terengganu's Rainfall Stations Based on Persistent Homology", Thai Journal of Mathematics, 197–211, (2022).

[3] A. S. Shakthi Aswin, R. U. Gobithaasan, Kelin Xia, Kenjiro T. Miura. "The Efficacy of Weight Control in Persistence Vectors Detecting Geometrical and Topological Features from Point Cloud", AIP Journal Proceedings, to appear.

[4] Peter Bubenik, Michael Hull, Dhruv Patel, and Benjamin Whittle, "Persistent homology detects curvature". Inverse Problems, **36**(2): 025008, (2020).

[5] Renata Turkeš and Guido Montúfar and Nina Otter, "On the effectiveness of persistent homology", 2206.10551, arXiv, (2023).

[6] John Ruskin, "The Elements of Drawing: In Three Letters to Beginners", https://www.gutenberg.org/files/30325/30325-h/30325-h.htm.

# Advancing Precision and Smoothness of Shape Preserving with Quintic Trigonometric Bézier Curve

Md Yushalify Misro
School of Mathematical Sciences, Universiti Sains Malaysia, 11800 Gelugor, Pulau Pinang, Malaysia.

## Abstract

This study integrates an optimization technique into positivity- and monotonicity-preserving interpolation methods to enhance curve smoothness by refining free shape parameters. These parameters play a pivotal role in defining curve geometry, granting users the flexibility to fine-tune the final shape. However, selecting them arbitrarily can compromise both aesthetics and accuracy, leading to undesired results. To address this challenge, an optimization-driven approach is introduced to systematically determine the optimal shape parameters. Within this framework, three smoothness metrics —arc length minimization, strain energy minimization, and curvature variation energy minimization— are employed. The resulting curves are analyzed and compared to assess their ability to preserve data while maintaining smoothness. The findings affirm that this method not only optimizes free shape parameters effectively but also surpasses conventional techniques in computational efficiency.

# Deployable auxetic surface structures: From optimized shape to detail design implementation

Kazuki Hayashi
Kyoto University

## Abstract

This study presents a streamlined design framework for deployable auxetic surface structures, taking advantage of discrete differential geometry. The process begins by defining basis vectors, informed by a prescribed plan, support locations, and load conditions, to modify the structural shape via Dirichlet energy minimization. Next, a gradient-based optimization algorithm explores the optimal shape to minimize the linear strain energy, adjusting the weights of the basis vectors with gradients that are analytically derived through the chain rule. The final step aims to materialize the optimized geometry using a double-layer auxetic surface structure incorporating kerf joints. This approach achieves the flexibility required for deployment while providing the necessary stiffness for in-service performance.

# Second-order infinitesimal mechanism for bifurcation analysis and folding path approximation of rigid origami

Kentaro Hayakawa
Nihon University

## Abstract

We investigate the kinematic bifurcation of rigid origami and approximate its folding path with polynomials through the second-order infinitesimal mechanism analysis of a truss model, the assemblage of the pin-connected bars. The motion of the model is constrained by the compatibility condition so that the bar length does not change. The second-order infinitesimal mechanism is obtained from the series expansion of the compatibility condition and its existence condition is the system of homogeneous quadratic equations. The bifurcated mechanisms of rigid origami correspond to the different solutions of the existence condition. In addition, we can use a solution to the existence condition for a polynomial approximation of the folding path of the truss model.

# Shape generation of free-form grid shells with polygonal panels

Jingyao Zhang
Kyoto University

## Abstract

This study addresses the shape generation of free-form grid shells with polygonal panels through two distinct approaches:

(a) For the generation of triangulated meshes with a predefined Gaussian curvature distribution, e.g., Figures 1 and 2, we introduce an efficient two-step method that integrates discrete Ricci flow and optimization techniques [1, 2]. The first step is to find the feasible edge lengths satisfying the predefined Gaussian curvature distribution, making use of circular packings. The second step is to embed these edge lengths into a three-dimensional space, by solving an optimization problem.

(b) For the generation of free-form planar meshes composed of polygonal panels, e.g., the planar quadrilateral mesh as shown in Figure , we propose a mechanical approach, modelling the mesh as a planar tensegrity structure. Self-equilibrated tensegrity units enable planarity of the panels, although this is not explicitly addressed as an objective in solving the form-finding problem.

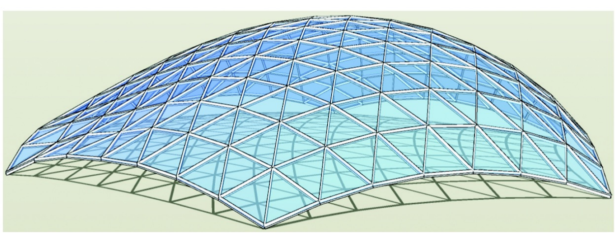


Figure 1: Surface with non-uniform Gaussian curvature

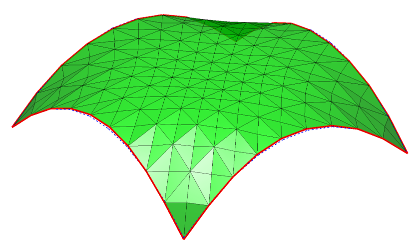


Figure 2: Globally developable surface

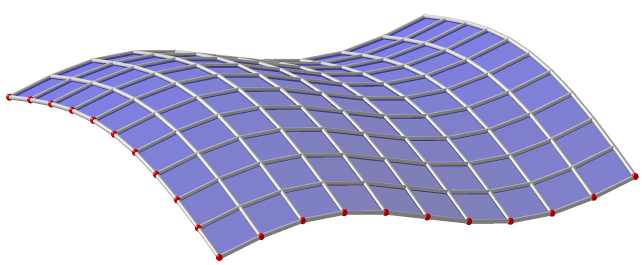


Figure 3: Planar quadrilateral mesh

## References

[1] J. Y. Zhang, M. Ohsaki, A design tool for globally developable discrete architectural surfaces using Ricci flow, Japan Architectural Review, Vol. 6 (1), 312410, 2023. 10.1002/2475-8876.12410

[2] S. Kaji, J. Y. Zhang, Finetuning discrete architectural surfaces by use of circle packing, Journal of Asian Architecture and Building Engineering, Vol. 23 (1), pp. 188–203, 2024.

# Tessellation as a design principle for mechanical metamaterials

Yusuke Sakai
Sony Computer Science Laboratories

## Abstract

Tessellation, a geometric pattern filling a plane without any gaps or overlaps, serves as a powerful tool for designing mechanical metamaterials. Mechanical metamaterials are artificial structures engineered for unique and tunable mechanical characteristics. In this talk, we introduce how simple polygonal tessellations can define the internal units of metamaterials, allowing tailorable mechanical responses through geometric design. By adjusting geometric configurations, we demonstrate intuitive tunability in deformation behaviors, leading to applications in transformable curved surfaces and tubular structures with unique mechanical behavior. Designing tessellation offers a systematic design scheme for adaptive and programmable structures, expanding possibilities for applications in aesthetic architectural roofs and mechanical devices.

# Surface generation for confidence-based data-driven computing in elasticity with application to reliability-based truss topology optimization

Yoshihiro Kanno
University of Tokyo

## Abstract

Data-driven computational elasticity is an emerging field of computational mechanics. This study presents a method predicting a bound for structural response, where the material responses are supposed to possess uncertainty. The uncertainty set is constructed by generating piecewise affine surfaces from a data set of material responses. We show that the problems for finding upper and lower bounds for the structural response can be recast as a mixed-integer linear programming problem, which can be solved globally with a branch-and-cut method. Then a fundamental property of the order statistics guarantees the confidence level for the probability that the obtained bound includes the structural response is no smaller than the target reliability. Furthermore, we discuss application to the reliability-based design optimization of truss structures is discussed.

# Optimization methods for continuum and latticed shells consisting of developable parts

Makoto Ohsaki
Kyoto University, Japan


## Abstract

To reduce the cost and time for construction of continuum shells and latticed shells for covering large architectural space, it is important to design the structures as an assembly of developable parts. For this purpose, this presentation summarizes the following three optimization methods developed as part of the JST CREST ED$^3$GE project:

1. A meshless and non-parametric two-level optimization approach is proposed for design of shell surfaces consisting of approximately developable patches. Developability is measured by the area of local Gauss map at the grid points. In the lower-level problem, the developability conditions are relaxed at some grid points to generate internal boundaries between approximately developable surface patches. In the upper-level problem, stiffness under the specified vertical loads is maximized. The design variables are the heights of the selected grid points, where developability conditions of some grid points are automatically relaxed. This way, a new class of structural shape optimization problem of shell surfaces consisting of piecewise developable surfaces is proposed to design shells with desirable geometrical characteristics in view of fabrication and construction.

2. In the design of latticed shell consisting of straight beams, it is important to have planarity of beam plates and surface panels while avoiding kinks at the joints. For this purpose, a hexagonal mesh consisting of straight beams connected at joints without torsion or kink is generated from Koebe mesh on a unit sphere obtained by spherical inversion in Möbius geometry. The parameters for Möbius transformation are optimized to obtain the latticed shell close to the target surface.

3. The cost and time for construction of gridshells consisting of quadrilateral meshes can be reduced by designing the shell as an assembly of planar beams. A gridshell with a planar quadrilateral mesh and planar curves is generated by discretizing an L-isothermic surface, where the directions of principal stresses coincide with the directions of principal curvatures under the uniform pressure load. The cross-sectional areas of gridshells are optimized to have the desired distribution of axial forces.


## References


[1] K. Hayakawa, M. Ohsaki and J. Y. Zhang, Meshless non-parametric shape design of piecewise approximately developable surfaces using discretized local Gauss map, J. Int. Assoc. Shell Spatial. Struct., Vol. 64, No. 1, pp. 5-14, 2024.

[2] K. Kabaki, K. Hayakawa, M. Ohsaki, Y. Jikumaru and Y. Yokosuka, Design of gridshells consisting of planar curves using Laguerre geometry, Proc. IASS Symposium 2024, Zurich, Switzerland, Int. Assoc. Shell and Spatial Struct., Paper No. 340, 2024.

[3] R. Watada and M. Ohsaki, Sequential generation method for hexagonal lattice shells with edge offset mesh, Proc. IASS Symposium 2024, Zürich, Int. Assoc. Shell and Spatial Struct., Paper No. 365, 2024.

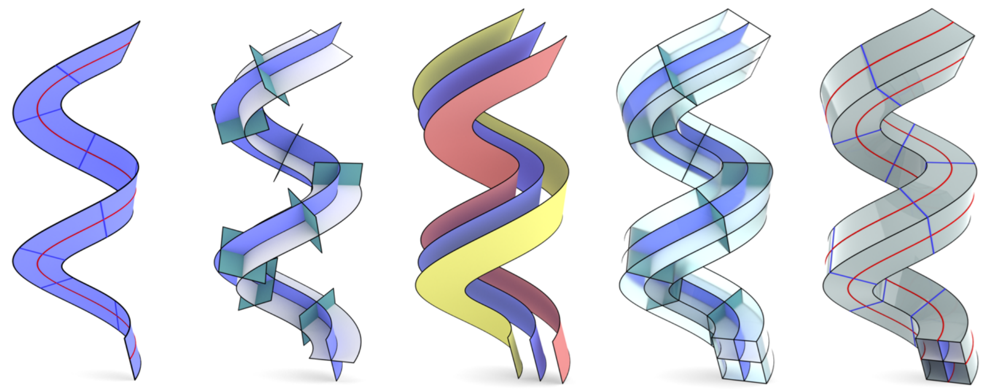
[4] M. Ohsaki, K. Hayakawa and J. Y. Zhang, Non-parametric structural shape optimization of piecewise developable surfaces using discrete differential geometry, Proc. Asian Congress of Structural and Multidisciplinary Optimization (ACSMO 2024), Zhengzhou, China, Paper B40314, 2024.

# All you need is rotation: Construction of developable strips – Part 1 Theory

Takashi Maekawa
Waseda University

Felix Scholz
Johannes Kepler University

**Abstract**



We present a novel method for generating developable strips along a space curve, offering flexible design. Central to this approach is the rotation angle, which governs the relationship between the Frenet frame of the input space curve and the Darboux frame of the curve on the resulting developable strip [1]. By treating this angle as a free design parameter, represented by any differentiable function along the curve, our method enables the creation of diverse developable geometries. This generalization significantly expands the design space, allowing for developable strips that share a common directrix curve. The rotation angle can be specified in various forms, such as constants, linear variations, sinusoidal patterns, or solutions to initial value problems defined by ordinary differential equations. By introducing this versatile framework, we advance the theoretical understanding of developable surface design, providing a powerful toolset for exploring and manipulating developable geometries with exceptional flexibility.

## References

[1] T. Maekawa and Felix Scholz, "All you need is rotation: Construction of developable strips", ACM Transactions on Graphics, vol. 43, no. 6, 2024.

# All you need is rotation: Construction of developable strips – Part 2 Applications

Takashi Maekawa
Waseda University

Felix Scholz
Johannes Kepler University

## Abstract




The versatility of the proposed method is demonstrated through both computational and physical examples, showcasing its broad range of applications. These include architecture, windmill blade design, curved folding, triply orthogonal structures, and the creation of surfaces with log-aesthetic curves. Such examples highlight the method's potential in fields like architectural design, industrial design, and papercraft modeling, offering a powerful tool for innovative surface design and fabrication. Specifically, we present:

- Architectural Design: A helical structure spanning the parameter range $0 \leq t \leq 2\pi$
- Inverted Catenary Arch: A model composed of two developable surfaces intersecting to form the shape of an inverted catenary.
- Deltoid Evolute: A construction based on the evolute of a deltoid curve, which intriguingly forms another deltoid when viewed from above. This is expressed through developable surfaces aligned along the deltoid.
- Papercraft Windmill Blade: We designed a vertical papercraft model with a developable surface. Unlike horizontal-axis turbines, vertical-axis turbines are wind-direction insensitive, removing the need for yaw control.
- Additional examples demonstrating the versatility of the method will be presented during the talk.

## References

[1] T. Maekawa and Felix Scholz, "All you need is rotation: Construction of developable strips", ACM Transactions on Graphics, vol. 43, no. 6, 2024.

# Isogeometric Analysis of Membrane and Cable Structures: A Design of Umbrella Zero-Stress State

Maya Okada / Naoyuki Fujita / Takuya Terahara / Yastoshi Taniguchi / Kenji Takizawa
Waseda University, 1-6-1 Nishi-Waseda, Shinjuku-ku, Tokyo, Japan

Tayfun E. Tezduyar
Rice University, MS 321, 6100 Main Street, Houston, TX 77005, USA

## Abstract

An umbrella is a common item that requires aesthetically and functionally good design. A wrinkle-free design is suitable in both directions, and for manufacturing reasons, zero-stress state (ZSS) of each membrane part is flat. We model an umbrella using T-splines, which we developed in [1], and using geometric knowledge [2] and steady-state structural mechanics. We use a newly developed Bézier simplex and combined T-splines to represent the membrane parts (see 1). To design the ZSS, we use the integration-point-based zero-stress state (IPBZSS) technique [3]. The bone parts are connected with the membrane with the method described in [1], and we newly developed the torsion representation (see 2 for a test) to stabilize the bone parts of the umbrella.

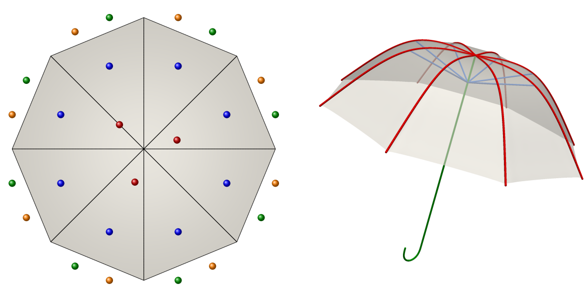


Figure 1: Simplex geometry with higher-order continuous and computational result

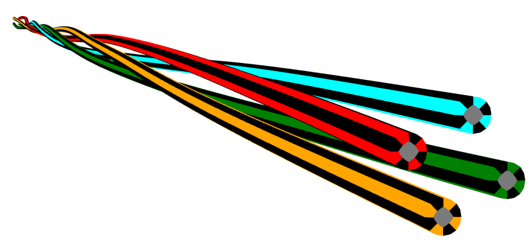


Figure 2: Isogeometric analysis of cable structure

## References

[1] T. Terahara, K. Takizawa, and T.E. Tezduyar, "T-splines computational membrane–cable structural mechanics with continuity and smoothness: I. Method and implementation", Computational Mechanics, **71** (2023) 657–675.

[2] T. Terahara, S. Nishikawa, A. Suzuki, K. Takizawa, and T. Maekawa, "Geometric modeling of umbrella surfaces", Computer-Aided Design, **175** (2024) 103750.

[3] T. Sasaki, K. Takizawa, and T. E. Tezduyar, "Aorta zero-stress state modeling with T-spline discretization", Computational Mechanics, **63** (2019) 1315–1331.

# Continuity and Smoothness in T-Splines Representations of Structures with Different Parametric Dimensions

Takuya Terahara / Kenji Takizawa

Waseda University, 1-6-1 Nishi-Waseda, Shinjuku-ku, Tokyo, Japan

Tayfun E. Tezduyar

Rice University, MS 321, 6100 Main Street, Houston, TX 77005, USA

## Abstract

We present a computational method using T-splines discretization for structural mechanics with different parametric dimensions are connected with continuity and smoothness. The Isogeometric analysis (IGA) gives accuracy to structural mechanics computations [1], and higher-order continuity allows use of the higher-order differential equations, such as the Kirchhoff–Love shells [2]. In IGA, connecting a 1D structure, such as a cable, to a 2D structure, such as a shell, is not that straightforward. That is because the control points are not on the cables or surfaces. The simple approach requires an extra refinement to have $C^0$ continuity functions that represents the position on the cables or surfaces. We proposed a new discretization method using T-splines [3, 4]. We present computations of test and parachute deformation. The computations demonstrate how the method works.

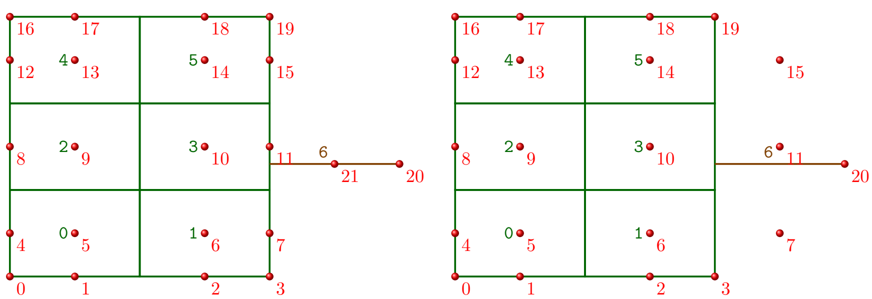


Figure 1: Membran–cable structures with $C^0$ and $C^1$ continuous

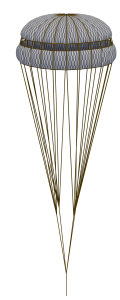


Figure 2: Parachute deformation

## References

[1] K. Takizawa, T. E. Tezduyar, and T. Terahara, "Ram-air parachute structural and fluid mechanics computations with the space–time isogeometric analysis (ST-IGA)", Computers & Fluids, **141** (2016) 191–200.

[2] Y. Taniguchi, K. Takizawa, Y. Otoguro, and T.E. Tezduyar, "A hyperelastic extended Kirchhoff–Love shell model with out-of-plane normal stress: I. Out-of-plane deformation", Computational Mechanics, **70** (2022) 247–280.

[3] T. Terahara, T. Kuraishi, K. Takizawa, and T.E. Tezduyar, "Computational flow analysis with boundary layer and contact representation: II. Heart valve flow with leaflet contact", Journal of Mechanics, **38** (2022) 185–194.

[4] T. Terahara, K. Takizawa, and T.E. Tezduyar, "T-splines computational membrane–cable structural mechanics with continuity and smoothness: I. Method and implementation", Computational Mechanics, **71** (2023) 657–675.

### Generation of Aesthetic Shapes: Klein Geometry, Integrability and Self-Affinity

Kenji Kajiwara
Institute of Mathematics for Industry, Kyushu University, Japan

Yoshiki Jikumaru
Faculty of Information Networking for Innovation and Design, Toyo University, Japan

Shun Kumagai
Hachinohe Institute of Technology, Japan

**Abstract**

In this talk, we consider a class of plane curves called the *log-aesthetic curves* (LAC) and their generalizations which have been developed in industrial design as the curves obtained by extracting the common properties among thousands of curves that car designers regard as aesthetic. We consider these curves in the framework of *similarity geometry* (Klein geometry associated with $CO^+(2,\mathbb{R}) \simeq SO(2) \ltimes \mathbb{R}^+$) and characterize them as invariant curves under the integrable deformation of plane curves governed by the *Burgers equation*. We propose a variational principle for these curves, leading to the stationary Burgers equation as the Euler-Lagrange equation[1, 3]. We then extend the LAC to space curves by considering the integrable deformation of space curves under similarity geometry. The deformation is governed by the coupled system of the *modified KdV equation* satisfied by the similarity torsion and a linear equation satisfied by the curvature radius. The curves also allow the deformation governed by the coupled system of the *sine-Gordon equation* and associated linear equation. The space curves corresponding to the travelling wave solutions of those equations would give a generalization of the LAC to space curves. We also consider the surface constructed by the family of curves obtained by the integrable deformation of such curves. A special class of surfaces corresponding to the constant similarity torsion yields quadratic surfaces (ellipsoid, one/two-sheeted hyperboloid and paraboloid) and their deformations, which may be regarded as a generalization of the LAC to surface. We discuss the construction of such curves and surfaces together with their mathematical properties, including integration scheme of the frame by symmetries, and present various examples of curves and surfaces.

Finally we discuss the *self-affinity* of plane curves that has been proposed in the area of industrial design as a characteristic property of the LAC. After some investigations and extending the definition[3], we propose a new class of "aesthetic curves" with self-affinity, which includes the logarithmic spiral (special case of the LAC) and quadratic curves (parabola, hyperbola and ellipse) under the framework of *equiaffine geometry* (Klein geometry associated with $SL(2,\mathbb{R})$). It may be an interesting problem to investigate the similar class of curves in Möbius geometry.

## References

[1] Jun-ichi Inoguchi,Kenji Kajiwara, Kenjiro T.Miura, Masayuki Sato, Wolfgang K.Schief and Yasuhiro Shimizu, Log-aesthetic curves as similarity geometric analogue of Euler's elasticae, Comp. Aided Geom. Design, **61** (2018) 1–5, https://doi.org/10.1016/j.cagd.2018.02.002.

[2] Jun-ichi Inoguchi, Kenji Kajiwara, Kenjiro T. Miura, Yoshiki Jikumaru and Wolfgang K. Schief, Log-aesthetic curves: similarity geometry, integrable discretization and variational principles, Comput. Aided Geom. Design **105**(2023) 102233, https://doi.org/10.1016/j.cagd.2023.10223.

[3] Shun Kumagai and Kenji Kajiwara, Self-affinities of planar curves: towards unified description of aesthetic curves, arXiv:2407.17008v1, https://doi.org/10.48550/arXiv.2407.17008, to appear in Japan J. Indust. Appl. Math. (2025).

# Geometry of Michell-Prager structures and hanging membranes

Yoshiki Jikumaru
Faculty of Information Networking for Innovation And Design, Toyo University, Japan
Kentaro Hayakawa
Department of Conceptual Design, College of Industrial Technology, Nihon University, Japan
Kazuki Hayashi
Department of Architecture and Architectural Engineering, Graduate School of Engineering, Kyoto University, Japan
Kenji Kajiwara
Institute of Mathematics for Industry, Kyushu University, Japan
Yohei Yokosuka
Department of Architecture and Architectural Engineering, Kagoshima University, Japan

**Abstract**

In this talk, we introduce some geometric objects motivated by the structures in architectural design. In the first part, we focus on so-called Michell-Prager-type structures. This is joint work with Yohei Yokosuka, Kazuki Hayashi, Kentaro Hayakawa, and Kenji Kajiwara [1]. Considering a quadrilateral mesh with such a structure on its diagonals, we can obtain the privileged discrete isothermic surfaces introduced by Bobenko and Pinkall. Their mechanical properties can be derived from the result by Schief. We also introduce the relation with the discrete log-aesthetic curves proposed in [3]. In the second part, we introduce the geometry of hanging membranes. This is joint work with Yohei Yokosuka [2]. We formulate the hanging membranes according to the classical shell membrane theory. Remarkably, the in-plane equilibrium condition can be characterized by the existence of a Combescure transformation of the membrane.

## References

[1] K. Hayashi, Y. Jikumaru, Y. Yokosuka, K. Hayakawa and K, Kajiwara, Parametric generation of optimal structures through discrete exponential functions: unveiling connections between structural optimality and discrete isothermicity. Struct. Multidisc. Optim. **67** 41 (2024). https://doi.org/10.1007/s00158-024-03767-1.

[2] Y. Jikumaru and Y. Yokosuka, Differential geometric formulation of hanging membranes: shell membrane theory and variational principle, Int. J. Math. Ind. **14** (2022), https://doi.org/10.1142/S2661335222500046.

[3] J. Inoguchi, K. Kajiwara, K. T. Miura, Y. Jikumaru and W. K. Schief, Log-aesthetic curves: similarity geometry, integrable discretization and variational principles, Comput. Aided Geom. Design **105**(2023) 102233, https://doi.org/10.1016/j.cagd.2023.10223.

# Geometric shape generation for ideal lighting

Yoshiki Jikumaru
Faculty of Information Networking for Innovation And Design, Toyo University, Japan

Kentaro Hayakawa
Department of Conceptual Design, College of Industrial Technology, Nihon University, Japan

Kazuki Hayashi
Department of Architecture and Architectural Engineering, Graduate School of Engineering, Kyoto University, Japan

Miyuki Koiso
Institute of Mathematics for Industry, Kyushu University, Japan

Shun Kumagai
Hachinohe Institute of Technology, Japan

### Abstract

In this talk, we propose a geometric shape generation of roof design for ideal lighting. The idea is based on the variational problem for anisotropic energy, originally proposed as a mathematical model for crystal growth. In our implementation, users can intuitively specify the direction in which they want to improve lighting. Moreover, our approach allows us to generate shapes with natural "internal boundary". While the main idea will be introduced in the smooth setting, we also propose a discretization for triangulated surfaces and shape generations.

Let $\mathbb{S}^2$ be the unit sphere in the 3-dimensional Euclidean space $\mathbb{R}^3$ and $\gamma : \mathbb{S}^2 \to \mathbb{R}$ be a positive-valued smooth function. For a smooth surface $M$ in $\mathbb{R}^3$, we define the *anisotropic energy* $\mathcal{F}_\gamma(M)$ as follows:

$$\mathcal{F}_\gamma(M) = \int_M \gamma(N)\, dA,$$

where $N$ denotes the unit normal vector field along $M$ and $dA$ denotes the area element. The minimizer of the anisotropic energy among all closed "surfaces" enclosing the same volume is called the *Wulff shape*. Moreover, a critical point of the anisotropic energy under volume-preserving variations can be characterized as a "constant anisotropic curvature" condition. If $\gamma \equiv 1$, the functional gives the area, and therefore, this situation can be regarded as a generalization of constant mean curvature (CMC) surfaces, which gives a mathematical model of soap bubbles. The above model of "generalization of soap bubbles" is useful for shape generation in the following two ways:

- While soap bubbles are "homogeneous", Wulff shapes, in general, "change the size of the faces according to the direction of the normal vector", which can be used to specify the "direction of lighting".
- While soap bubbles have "no edges", Wulff shapes generally have "edges", and natural "internal boundaries" can be generated without connecting several surface pieces.

## References

[1] Miyuki Koiso and Bennett Palmer, Geometry and stability of surfaces with constant anisotropic mean curvature, Indiana University Mathematics Journal **54** (2005), 1817–1852.

# Pillow boxes as developable surfaces with curved foldings

Miyuki Koiso
Institute of Mathematics for Industry, Kyushu University, Japan

### Abstract

## References

# Intrinsic and extrinsic singularities and curvatures of piecewise smooth surfaces

Miyuki Koiso
Institute of Mathematics for Industry, Kyushu University, Japan

### Abstract

## References

# Interactive Design and Efficient Simulation of Developable Surfaces with Curved Folds

Jun Mitani
University of Tsukuba, Japan

## Abstract

Developable surfaces with curved folds are widely used in engineering, architecture, digital fabrication, and computational design. However, their strict geometric constraints make both modeling and simulation non-trivial tasks. This talk introduces two complementary research approaches that address these challenges by focusing on interactive 3D modeling and efficient crease pattern-based simulation.

The first approach, proposed by Mitani and Ohashi [1], presents an interactive 3D modeling framework that enables users to directly manipulate curved fold structures in three-dimensional space. This method introduces a novel user interface based on a *handle curve*, which serves as an auxiliary control element for shaping the developable surface. By specifying both a crease curve (the curved fold) and a handle curve, users can intuitively deform the surface while ensuring developability and avoiding ruling collisions. This technique provides direct control over the 3D shape, making it particularly useful for interactive design applications in CAD modeling and digital fabrication.

In contrast, the second approach, developed by Sasaki and Mitani [2], focuses on efficiently generating 3D folded structures from 2D crease patterns. Instead of direct 3D manipulation, this method takes a given crease pattern, approximates curved folds using polylines, and applies a ruling-aware triangulation to construct a 3D model that accurately simulates the folded state. Implemented in a web-based Origami Simulator [3], this approach enables fast and computationally efficient simulation, making it ideal for applications where the input is a crease pattern rather than a predefined 3D model. The method allows for quick evaluation of different folding scenarios and helps designers explore complex curved fold structures without manual 3D adjustments.

By integrating these two methods—interactive modeling and efficient simulation—we provide a powerful framework for designing and analyzing developable surfaces with curved folds. This talk will discuss the theoretical foundations, algorithmic implementations, and potential applications of these techniques in digital fabrication, CAD modeling, and origami design.

## References

[1] Jun Mitani, Kaoru Ohashi, "Interactive Curved Fold Modeling using a Handle Curve", Computer-Aided Design and Applications, **20(2)** (2022) 275–289, https://doi: 10.14733/cadaps.2023.275-289.

[2] Kosuke Sasaki, Jun Mitani, "Simple implementation and low computational cost simulation of curved folds based on ruling-aware triangulation", Computers & Graphics, **102** (2021) 213–219, https://doi:10.1016/j.cag.2021.09.012.

[3] Amanda Ghassaei, Erik D. Demaine, Neil Gershenfeld, "Fast interactive origami simulation using gpu computation", Origami **7** (2018) 1151–66.

# Modeling of Discrete Developable Surfaces with a Break Using Trace Diagrams on the Gaussian Sphere

Kosuke Horiuchi
University of Tsukuba, Japan

Jun Mitani
University of Tsukuba, Japan

### Abstract

In recent years, industries such as manufacturing and architecture have increasingly adopted CAD software for shape modeling and product design. This shift minimizes the cost and effort associated with physical prototyping. Despite advancements, designing developable surfaces while maintaining intuitive and precise interfaces remains a challenge. Developable surfaces, characterized by zero Gaussian curvature, are created by twisting or bending unstretchable sheet materials. They are represented through a trajectory of straight line elements called rulings. One remarkable technique is "Non-Crease", which generates complex curvature without traditional folds by creating indentations called breaks, which are degenerated creases with zero length. By enabling the computational design of Non-Crease surfaces, it is expected to facilitate the digital archiving, analysis, and creation of art pieces.

This research aims to support the design of developable surfaces with a break by proposing an interface that integrates Gauss sphere-based trace diagrams[1]. These diagrams map the behavior of surface normal vectors onto the Gauss sphere, aiding in the visualization of curvature distribution around vertices. A key property of trace diagrams on the Gauss sphere for developable surfaces is that the areas enclosed by the traces sum to zero. By editing these diagrams, users can intuitively create and modify developable surfaces with breaks.

The methodology involves starting with a predefined template for developable surfaces with a break. The trace diagram corresponding to this template is visualized and editable. Users adjust trace lengths and angles to create their desired shapes, with the system performing optimizations to ensure the areas enclosed by traces on the Gauss sphere sum to zero, a key constraint of developable surfaces. Trace intersections and the enclosed areas are calculated in real-time to guide this process. Post-editing, the interface generates a crease pattern. Finaly, crease pattern is validated 3D shape and physical realization, supported by Origami Simulator[2].

Results show that this interface enables effective control over the ruling angles and the creation of various developable surface shapes. Optimizations minimize area discrepancies in trace diagrams, enhancing the accuracy of the resultant designs.

Future work will address extending the system to handle shapes with multiple convex and concave regions, improving usability, and reducing optimization errors. This study highlights the potential of trace diagram-based modeling as a powerful tool for designing intricate and mathematically accurate developable surfaces.

## References

[1] David Huffman, "Curvature and Creases: A Primer on Paper", IEEE Trans. on Computer, Vol. C-25, No. 10, pp. 1010–1019, 1976.

[2] Amanda Ghassaei, Erik D. Demaine, Neil Gershenfeld, "Fast, Interactive Origami Simulation using GPU computation", Origami, Vol. 7, pp. 1151–1166, 2018.

# Parametric Design Tools for 3D Curved-Origami Shapes in Conceptual and Prototype Architectural Design

Aida Safary
University of Tsukuba, Japan

Jun Mitani
University of Tsukuba, Japan

### Abstract

In this research, we produce digital parametric tools of 3D origami-based architectural elements, enabling the users to modify and manipulate basic geometrical features of these tools to explore and create extended geometric variability options of the mentioned structures. In our first project, we designed a module for the One-Fold project by Patkau architects with specific options for changing the shape parameters to give users the ability to generate various structures of architectural shelters of the same 3D shape. The One-Fold project consists of a rectangular or square plane with a single fold as of its diagonal line, which creates conic curve borders when folded [1]. As our second research project, we developed a digital system for the parametric design of David Huffman's design with ellipses of two-degree two-vertices. In this design tool, we apply parameters for changing the fold angle, the size of the structure, and the rotation of curved lines inside our 3D shape. In this design tool, we applied an approach similar to the additive algorithm method for generating our shape step by step as a quad mesh structure [2, 3]. In our future research project, we intend to compare our digital tools with physical prototypes using 3D scanners, evaluate the Elastica curves of both models and use the RMSE method for surface error analysis.

## References

[1] Aida Safary, Hamid Shafieasl, and Jun Mitani, "Geometric design tool for One-Fold, a curved origami with a single fold", J. Geom. Graph., **28** (2024) 89–101.

[2] Erik D. Demaine, Martin L. Demaine, and Duks Koschitz, "Reconstructing David Huffman's legacy in curved-crease folding", Origami **5** (2011) 39–52.

[3] Levi H. Dudte, Gary P. T. Choi, and L. Mahadevan, "An additive algorithm for origami design", Proc. Natl. Acad. Sci., **118** (2021) e2019241118.

# Hoberman's Scissor Mechanism and Digital fabrication

Higa Miyashiro Pamela
University of Tsukuba, Japan

Yiyang Jia
Japan Women's University, Japan

Mitani Jun
University of Tsukuba, Japan

## Abstract

The Hoberman mechanism, developed by Chuck Hoberman, is renowned for its applications in toys and architecture. The Hoberman Sphere, a collapsible toy, exemplifies the mechanism's versatility through its use of circular elements with scissor-like linkages to expand and contract. These linkages are fundamental to the mechanism's deployment, which is also evident in architectural applications, most notably in the Hoberman Arch, showcased at the 2002 Winter Olympics. This structure, composed of segmented arcs arranged in multiple layers, highlights the scalability and adaptability of the mechanism. Our research on the Hoberman mechanism spans geometric principles, movement profiles, and deployment constraints [1, 2, 3]. In this study, we combine theoretical research, digital modeling, and physical prototyping. Using tools such as Rhinoceros and Grasshopper, we facilitated digital simulations of scissor mechanisms. Additionally, we employed digital fabrication techniques to create physical prototypes. We tested three configurations: an irregular polyline, a regular dodecagon, and a circumscribed irregular dodecagon. These models allowed us to explore how geometry impacts movement and to address the challenges posed by irregular configurations. We focused specifically on the irregular dodecagon model, as its closed and irregular geometry highlights the constraints of the method. By maintaining proportional link lengths and angular relationships, we ensured proper deployment of the mechanism. A physical prototype of the irregular dodecagon was fabricated, and its movement matched the predictions from the simulations. Inspired by the Hoberman Arch, which incorporates multiple layers of Hoberman mechanisms into a discretized semicircle, we added additional layers to the irregular dodecagon model. However, the multi-layered design exhibited resistance and deformation during movement, suggesting the presence of over-constraints in the model. To further investigate, we simulated the movement of the double-layered irregular dodecagon. The simulation, combined with an in-depth geometric analysis, revealed that multi-layered designs using irregular polygons inherently lead to over-constraints. Overall, our study demonstrates the Hoberman mechanism's adaptability in toys, architecture, and deployable structures. However, we also identified limitations in the shapes that can successfully support multi-layered Hoberman mechanisms, particularly in irregular configurations.

## References

[1] Gomez, Alfonso, "Deployable Domes Based on Angulated Scissors: A Method of Design Based on Geometrical Construction", The International Journal of Designed Objects, **9** (2015) 1.

[2] Asefi, Maziar, Ebadia, Atefeh, and Ghasemib, Azam, "Geometry feasibility of angulated scissor-like elements in a constant perimeter", (2019).

[3] Sun, Xuemin, Yao, Yan-An, and Li, Ruiming, "Novel method of constructing generalized Hoberman sphere mechanisms based on deployment axes", Frontiers of Mechanical Engineering, **15** (2020) 89–99.

# Curved Surface Structures with Excellent Mechanical Rationality and Constructability/Fabricability

Yohei Yokosuka
Kagoshima University, Japan

## Abstract

There are geometrically defined classes of surfaces and curves suitable for each surface shape, such as membrane structures that resist tensile stress and shell structures that resist compressive stress. These surface and curve classes are categorized as mechanically motivated and those motivated by constructability and member fabrication. From a mechanical point of view, F. Otto uses an extremely minimal curved surface for the membrane structure and H. Isler applied a suspended curved surface for the shell structure, designing a curved surface structure that is mechanically rational. F. Candela uses HP curved surfaces in curved structures, designing curved structures that are superior from constructability. In recent years, there has been progress in research on the design of curved surface structures that rediscover the properties of both the mechanical and constructional perspectives from geometry by means of discrete differential geometry. This presentation will introduce a class of surfaces and curves that can be applied to curved structures in architecture.

# Lie Spherical Geometry and Design of Curved Surface Structures

Yohei Yokosuka
Kagoshima University, Japan

## Abstract

The purpose of this study is to employ discrete surfaces as shape elements and to construct a design method for curved surface structures by parametric deformation using Lie spherical transformations. NURBS surfaces and Bézier surfaces are useful parametric surface generation methods as surface design tools. However, the properties of the cross ratio and the developability of the surfaces covered by the coordinate lines are not preserved. Lie spherical geometry can perform Lie spherical transformation, which maps curvature line coordinates to curvature line coordinates. Curvature line coordinates can be represented by discrete surfaces filled with circles on the surface; the Möbius transform, one of the Lie spherical transforms, allows transformations that preserve the cross ratio, and isothermal coordinates can be mapped to isothermal coordinates. This presentation will describe the method of generating 3-D and 2-D surfaces and the mechanical properties of isothermal coordinates.

# Form-finding of Composite Tensile Structures by Finite Element Technique based on Nodal Coordinate Assumption

Yohei Yokosuka
Kagoshima University, Japan

## Abstract

In general, the finite element method used in structural analysis uses a finite element method assumed displacement in which the displacement of a node is formulated as an unknown function. On the other hand, the finite element technique based on nodal coordinate assumption can formulate the coordinates of nodes themselves as unknown functions and perform stress-deformation analysis and form-finding analyses. In this presentation, I derive a virtual work equation using embedded coordinates and explain the differences in strain and stress derived from the equilibrium equation after deformation. In the equilibrium equation, the displacement assumption corresponds to the first Piola-Kirchhoff stress tensor and the coordinate assumption corresponds to the Cauchy stress tensor. Therefore, when the before deformed configuration is used as the reference configuration, proposed method presents a natural formulation and is suitable for nonlinear analysis using the total Lagrange method. In addition, the form-finding analysis of composite tensile structures with beam, truss, and membrane elements using the nodal coordinate assumption based on finite element technique will be explained.

# Temporary structures with curved folding

Yohei Yokosuka
Kagoshima University, Japan

## Abstract

Temporary housing needs to provide a large number of houses quickly after a disaster, so it is useful to have temporary structures using curved folds that are superior in space-saving stocking, portability, and quick construction, and can immediately expand flat plates into a three-dimensional structure. Koiso et al. derive an explicit expression for the maximum volume solution of the pillow box and show that the bottom curve of the pillow box is an elastic curve. In this presentation, a scaled experimental model of a temporary structure with curve folding is fabricated to show that curve folding is possible with rigid body deformation. Furthermore, an example of numerical analysis is shown where the generation of a curved surface shape that is the solution to the maximum volume of a pillow box is linked to structural analysis and applied to a multi-objective optimization problem where the volume evaluated as architectural planning performance and the maximum displacement evaluated as structural performance are used as indices.

# Extension of $\kappa$-curve

Kenjiro T. Miura
Graduate School of Science and Technology, Shizuoka University, Japan

### Abstract

The $\kappa$-curve[1, 2] is a recently published interpolating spline which consists of quadratic Bézier segments passing through input points at the loci of local curvature extrema. We extend this representation to control the magnitudes of local maximum curvature in a new scheme called *extended*- or $\epsilon\kappa$-curves.

$\kappa$-curves have been implemented as the curvature tool in Adobe Illustrator® and Photoshop®, and are highly valued by professional designers. However, because of the limited degrees of freedom of quadratic Bézier curves, it provides no control over the curvature distribution.

We propose new methods that enable the modification of local curvature at the interpolation points by degree elevation of the Bernstein basis as well as application of generalized trigonometric basis functions. By using $\epsilon\kappa$-curves, designers acquire much more ability to produce a variety of expressions, as illustrated by our examples.

## References

[1] Yan, Z., Schiller, S., Wilensky, G., Carr, N., Schaefer, "$\kappa$-curves: Interpolation at local maximum curvature", ACM Transactions on Graphics **36**(4), Article 129 (2017).

[2] Yan, Z., Schiller, S., Schaefer, S., "Circle reproduction with interpolatory curves at local maximal curvature points". Computer Aided Geometric Design **72**(6), 98–110 (2019).

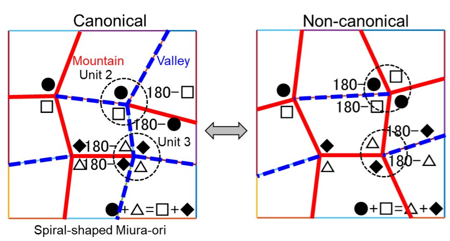
# Geometric Shape Generation by Singular Generalized Miura-ori with Canonical and Non-canonical Arrangements

Hiroyuki Tagawa
Department of Architecture, Mukogawa Women's University, Japan

## Abstract

Generalized Miura-ori with the canonical arrangement, including well-known Miura-ori as well as proposed arc- and spiral-shaped Miura-ori [1], can be folded flat without causing self-intersections. A total of 26 patterns of singular generalized Miura-ori, which is defined as the generalized Miura-ori that has symmetry and regularity in included angles to enable rigid flat-foldability with linked one degree-of-freedom motion, is counted for the canonical arrangement as follows: 11 patterns for $K_1 = K_4$, $K_2 = K_3$, 11 patterns for $K_1 = K_2$, $K_3 = K_4$, and 4 patterns for $K_1 = K_2 = K_3 = K_4$ [2]. Among 26 patterns, the arc- and spiral-shaped Miura-ori, which are classified as $K_1 = K_2$, $K_3 = K_4$, are the only patterns in which all the fold lines are not parallel to each other.

Non-canonical arrangement is obtained by exchanging the included angles at the diagonal positions of the Units 2 and 3 in the canonical arrangement and accordingly changing the mountain and valley folding directions as shown in Fig. 1. These exhibit 3D cylindrical or vault shape while satisfying the linked folding conditions. A total of 17 patterns of singular generalized Miura-ori is counted for the above non-canonical arrangement as follows: 11 patterns for $K_1 = K_4$, $K_2' = K_3'$, 2 patterns for $K_1 = K_2'$, $K_3' = K_4$, and 4 patterns for $K_1 = K_2' = K_3' = K_4$. Huffman tessellation and the dual of Miura-ori (Hourglass mode) are classified as $K_1 = K_2' = K_3' = K_4$. Cylindrical closed shape is obtained by the optimization on included angles of the quadrilaterals and deployment angle as shown in Fig. 2.




$$(K_4 - K_3)(1 - K_1K_2) = (K_1 - K_2)(1 - K_3K_4)$$

$$K = \frac{1 + \cos\theta_i \cos\theta_j}{\sin\theta_i \sin\theta_j}$$

$$K' = \frac{1 + \cos(\pi - \theta_i)\cos(\pi - \theta_j)}{\sin(\pi - \theta_i)\sin(\pi - \theta_j)} = K$$

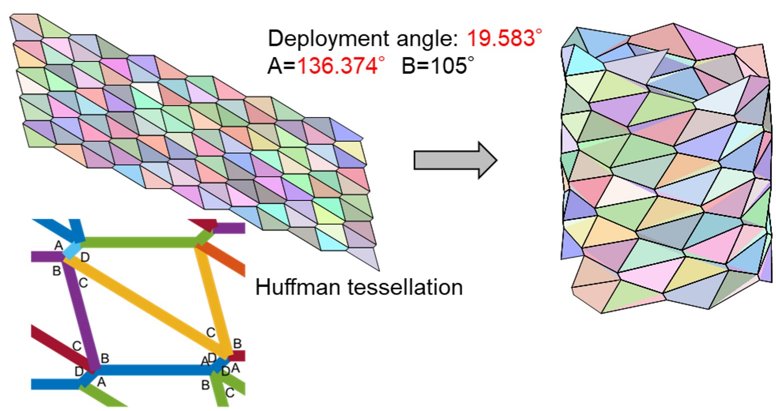



Figure 3: *

Fig. 1 Dual conversions

Fig. 2 Cylindrical closed shape obtained by optimization

## References

[1] H. Tagawa, N. Yoshioka, T. Suzuki, "Proposal of arc- and spiral-shaped Miura-ori and its application to the design of large roof architecture", Proceedings of the IASS symposium 2022, Beijing, China, 2022.

[2] H. Tagawa, A. Sugimura, A. Mukai, K. Inomata, "Counting-up and classification of all combination patterns of singular generalized Miura-ori", Proceedings of the IASS symposium 2024, Zurich, Switzerland, 2024.